

## An Improbable Story

Pierre and Jacques seated themselves on the metro. Pierre, as was his habit, took the window seat and opened the window wide. He loved the feel of wind on his face, and the view of the countryside speeding by. In his student years he used to ride a big motorcycle, traveling at tremendous speeds on mountain paths, just to get this sensation. But that was years ago. When they received their professorships, his old friend Jacques had convinced him that this recreation was hardly fitting for a venerable professor of probability and statistics. Jacques was probably right, but his frequent contact with staid lawyers, being a professor of business administration, probably overly influenced him. Now all that was left was to take the window seat on the speedy metro during their monthly trips into the country.

But Pierre did not always give in so easily. Just before boarding the train he had managed to convince Jacques to buy a lottery ticket. Jacques was reticent at first, not wanting to squander his meager professor's salary on such a low probability of winning venture. Pierre knew as well as Jacques, probably even better, how small the chances were of winning anything, but he bought a ticket for the sport, and convinced his old friend to do so as well. The excitement was at its peak since the results were to be announced this very morning, indeed they purchased the last two tickets sold before the official closing of the lottery office. They would be able to hear the results on his pocket radio, which picked up the radio station for most of the way.

The real reason for his interest in the lottery this year, was that the new rules intrigued him. As usual the jackpot amount was not announced. That was the usual trick, enheightening the excitement since every year the prize became unpredictably higher. But this year they had added a twist, there would be two winners sharing the jackpot; and rather than dividing it equally, the first winner would receive two thirds of the jackpot, and the runner-up one third. To make the tension truly unbearable, the two co-winners would be chosen and announced first, and only afterwards would the final decision be made – which of the two would receive double the other's share.

During the first hour of travelling their conversation covered many topics. They consciously avoided mentioning the lottery until the designated hour. Then with elegance Pierre switched on his radio and the two comrades strained to hear the news report through the static. Then came the moment the entire country had been waiting for. The two winners were —

Pierre and Jacques glanced at their lottery tickets, and simultaneously

their mouths dropped open. Miraculously the two winning numbers were riding side by side in the same train cabin. ‘But that is practically impossible’, Jacques exclaimed, unbelievably. ‘Certainly highly improbable’, agreed Pierre, trying to reconcile the facts as he now understood them with everything he professionally knew. The two could do nothing but numbly stare at each other during the commercial break.

Then the announcer’s voice returned. It was becoming more and more difficult to understand him through the growing volume of static. They could hear him explain once again the new rules. And then he started reading off, ‘and the runner-up is’.

But at that moment the train rounded a hill and the radio station was totally lost. It would be a while before they would start picking up the next station, and there was nothing the friend could do for now. Nothing, that is, save to hold on tightly to the winning tickets.

‘Pierre’ Jacques restarted the conversation, ‘the tension is simply unbearable. Why don’t we agree to split the total jackpot equally between us. At this point neither of us knows which is to be richer. Wouldn’t that be the reasonable thing to do?’

‘Well I was just thinking about our predicament, and I must admit that I’m a bit puzzled’, replied Jacques. ‘You see I have an even better idea for you, one that will probably increase your share – do you want to hear about it?’

‘I thought so. Well let’s assume for concreteness that your share of the prize comes out to be 100,000. Then according to the rules, mine must be either twice as much - 200,000 or half as much - 50,000. Now since we have no idea which, we can assume that the probability of each of these cases is 50 %. Now I ask you, if I offered to swap tickets with you – would you agree?’

Pierre, realizing what his probabilistic friend was implying, did a quick calculation in his head. If he *did* agree to the swap he would with 50% probability win 200,000 and with 50% probability 50,000. So on the average he would stand to win  $1/2 ( 200,000 + 50,000 ) = 125,000$ . If he refused the switch, he would remain with only 100,000.

‘All right’, he replied at last with a grin on his face, ‘I expect to win 25,000 if I swap with you – but why do you want to swap?’ ‘Isn’t it obvious?’, Jacques replied in his professor’s lecturing tone, ‘I stand to gain the same amount by the swap. That’s the whole beauty of the idea, we both win!’

The swap was carried out on the spot, and the two friends sank back into silence. This time it was Pierre who broke into Jacques thoughts. ‘Assuming that my original share was 100,000 I now have (on the average of course)

125,000. Thus you must have either 250,000 or 62,500, with equal probability. Thus a further swap, if you would agree to one, would bring me  $1/2 (250,000 + 62,500) = 156,250$ , increasing my share by a further 31250 (with the lottery paying for it). What do you think?’

Jacque didn’t reply verbally this time, but merely winked and extended his ticket. ‘The next swap’, he answered with the certainty of someone who has already thought it all through, ‘will bring me a further 25 % winnings, not on the original amount but with compound interest. 195,312.50 ... 244,141 ... 305,176 ... 381,470 ...’

The two friends started swapping the tickets back and forth, faster and faster, imagining their winnings rising unboundedly, until after they had already surpassed 100 million each when a gust of wind blew the tickets from their hands and out the window.

The smiles were immediately erased. The friends realized that they were back to their previous economic state, a thought which they found hard to grasp. They had only become unimaginably rich a moment ago, but they felt as if they had been the richest men in the country for years.

‘You know’, said one of them, ‘they probably didn’t have enough money to pay us that much anyway.’ The train rounded a hill and the pocket radio came back to life. The friends heard, ‘and we will now repeat the winning ticket numbers. The first prize ...’. Simultaneously the two friends picked up the receiver and tossed it out the window.