$$H(z) = G\frac{\prod_{i=1}^{m}(z-\zeta_i)}{\prod_{i=1}^{n}(z-\pi_i)}$$

$$H(\omega) = G\frac{\prod_{i=1}^{m}(e^{j\omega}-\zeta_i)}{\prod_{i=1}^{n}(e^{j\omega}-\pi_i)}$$

$$|H(\omega)| = G\frac{\prod_{i=1}^{m}|e^{j\omega}-\zeta_i|}{\prod_{i=1}^{n}|e^{j\omega}-\pi_i|}$$

$$\angle H(\omega) = \sum_{i=1}^{m}\angle(e^{j\omega}-\zeta_i) - \sum_{i=1}^{n}\angle(e^{j\omega}-\pi_i)$$

12.2 DSP Flow Graphs

- $y = x$
- $y = f(x)$
- $z = g(y) = g(f(x))$
- $y = ax$
- $y = z^{-1}x$
- $y = z = x$
- $z = x + y$
- $z = x - y$
- $z = xy$
- $y_n = \Delta x_n = x_n - x_{n-1}$

- $y_n = a_0 x_n - a_1 x_{n-1}$

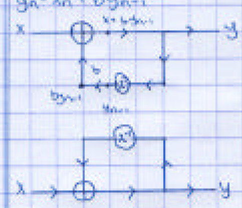
- $y_n = \sum_{k=0}^{N-1} a_k x_{n-k}$

Tapped Delay-line

- Multiply-And-Accumulate (MAC)

כמו קודם ניתן לממש זאת ע"י 2 מבנים, המקבילים למבנים של Feedforward.
המבנה הFeedback כעת:

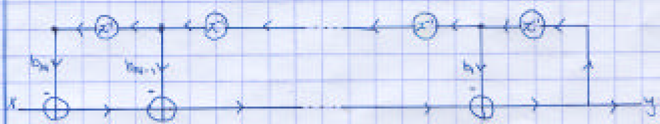
- $y_n = x_n + b y_{n-1}$

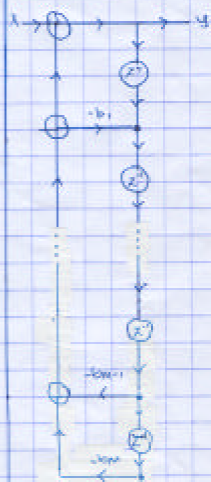


- $y_n = x_n - \sum_{m=1}^{M} b_m y_{n-m}$

המבנה All-pole (AR)
מימוש בעזרת MAC

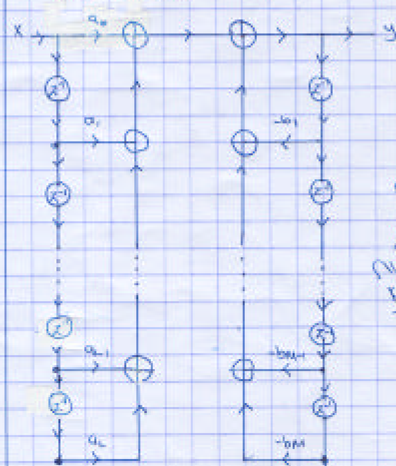
זה סדר ערך הפוך
ההגדרה.





- $y_n = \sum_{l=0}^{L} a_l x_{n-l} - \sum_{m=1}^{M} b_m y_{n-m}$

פילטר ARMA



ניתן להראות כי פילטר ARMA ניתן לפירוק לשני מסננים MA ו-AR ברצף. המסנן FIR ו- all-pole שלו יכולים להיות שונים בסדר. כאשר ניתן לבצע את המסננים ה-MA ו-AR אין צורך להיות בסדר זהה. הם קומוטטיביים, על שילוב חלוקות בין המסננים. אזי מותר לעשות המקבילים מאחד להגיע modified לשלב את שני:

ההבדל המהותי ניתן לראות את ההבדל בזמן מימוש, כמו, בפרקטיקה, המספר של אלמנטים החשובים הם לא נכונים. ה-feedback גורם loops של כל, בלבד מונחה, מהירות, וכו' גם, וכן הלאה הוא יותר בעייתי

12.3 מניפולציות אלגו' ב-DSP

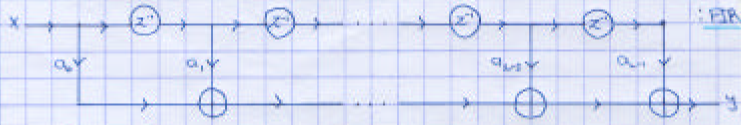
[illegible] DSP (גם FIR)
[illegible]

Symmetries: [illegible]
(1) [illegible]
(2) 'like signal merging' - [illegible]
(3) 2 [illegible] f ו-g [illegible] 'commute' [illegible]

[illegible] commute [illegible] DSP
• [illegible] ARMA [illegible] AR [illegible] MA [illegible]
[illegible] 'normal' [illegible]
[illegible]
(4) [illegible]
(1) [illegible]
(2) [illegible] T [illegible]
(3) [illegible] T [illegible]
(4) [illegible] input [illegible] output.
(5) hierarchical flow graphs: [illegible]
[illegible] Symmetries [illegible]
[illegible]

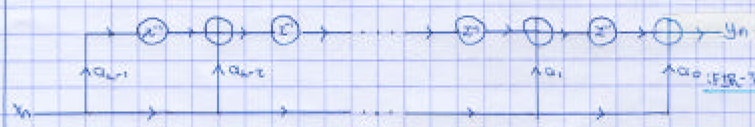
15.3 FIR Structures (graphical implementations)

FIR-Direct Form:
(1) [illegible]



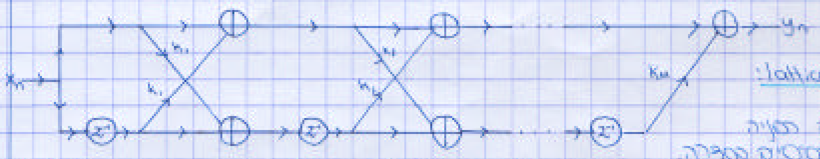
[illegible] MAC (multiply and accumulate) [illegible] MACs [illegible] DSP
[illegible] benchmark [illegible] DSP

FIR-Transposed Form:



[illegible]
[illegible] overflow errors [illegible] round-off noise [illegible]
[illegible] output [illegible] input [illegible]
[illegible] transposed [illegible]
[illegible] direct [illegible]
[illegible]

cascaded subfilters, [illegible] (עמ' 580-582)



lattice structure:

יתרונות: יציבה
[illegible]
[illegible] numerically robust [illegible]
[illegible] FIR [illegible]
IIR [illegible] ARMA [illegible]
[illegible] ([illegible]) [illegible]
[illegible]

14 מעבד DSP

מעבדי DSP מאופיינים ע"י ה-features הבאים: (1) [illegible] (MAC [illegible])
(2) [illegible] (3) zero-overhead loops - [illegible]
[illegible] (4) instruction pipelines (5) multiple memory buses and banks
(5) fast interrupt servicing (fast context) (6) [illegible]
(7) special addressing modes (כגון bit reversal).
[illegible] MAC [illegible]
[illegible] FT [illegible] MAC

14.1 MAC: [illegible]
[illegible] loop: [illegible]

loop
update i, update j
a ← a + $x_i y_i$

[illegible] x ו-y [illegible]
x_i ← [illegible] x_n [illegible] y_i [illegible] ([illegible] offset [illegible])
2 pointer registers (i, j) [illegible]
[illegible] 2 floating point or fixed arithmetic registers (x, y) [illegible]
[illegible] double-length register (z) [illegible] accumulator (a)
[illegible] loop [illegible] MAC:
[illegible] CPU [illegible]
[illegible] CPU [illegible]
[illegible] CPU [illegible]
[illegible] MAC [illegible]
[illegible] MAC augmented [illegible]
[illegible]

(1) update pointer to x_i
(2) update pointer to y_j
(3) load x_i into register x
(4) load y_j into register y
(5) fetch op(×)
(6) decode op(×)
(7) z ← x·y
(8) fetch op(+)
(9) decode op(+)
(10) a ← a + z

[illegible] DSP [illegible]
x ו-y [illegible]
[illegible] 2 address registers:
[illegible]
[illegible]

(1) update pointer to x_i
(2) update pointer to y_j
(3) load x_i → x
(4) load y_j → y
(5) fetch op (MAC)
(6) decode op (MAC)
(7) MAC: a ← a + x·y

(1) update pointer to x_i || update pointer to y_j
(2) load x_i → x
(3) load y_j → y
(4) fetch op (MAC)
(5) decode op (MAC)
(6) MAC: a ← a + x·y

14.2 ארכיטקטורת זיכרון

כדי לחשב את x_i ו-y_i של המכפלה הפנימית [illegible]. כיון שהם [illegible] [illegible] data [illegible] : 2 databases.
[illegible] buses [illegible] [illegible]
[illegible] multiple memory banks [illegible] buses [illegible]
[illegible] 5 שלבים [illegible] : (2) load $x_i \to x$ || load $y_i \to y$

[illegible] fetch ו-decode [illegible] CPU-ה [illegible]
ארכיטקטורת הרווארד: memory [illegible] data [illegible] fetches
[illegible]
(1) update pointer to x_i || update pointer to y_i
(2) load x_i to register x || load y_i to register y
(3) MAC: $a \leftarrow a + x \cdot y$

14.3 Pipelines

update 1	update 2	update 3	update 4	update 5		
	load 1	load 2	load 3	load 4	load 5	
		MAC 1	MAC 2	MAC 3	MAC 4	MAC 5

pipelining of a MAC:
[illegible] pipeline [illegible] DSP-ה
[illegible] MAC [illegible]
[illegible]
[illegible] pipeline-ה [illegible] (fetch, decode, execute) 3
T - [illegible] N - [illegible] D - [illegible]: • $T = N + D - 1$
DSP-ה [illegible] pipeline [illegible]

14.4 Interrupts, Ports

context switch: [illegible]
[illegible]
interrupt: [illegible]
port - [illegible] interrupt [illegible] ISR - Interrupt Service Routine
DSP-ה [illegible] zero-overhead context switching - ו interrupt [illegible]
[illegible]

ports: [illegible]
(1) serial - [illegible] baud rate [illegible]
(2) [illegible] 8
[illegible] Direct Memory Access (DMA) [illegible]
[illegible]
(3) [illegible]

14.5 Fixed and floating point

[illegible] Fixed point DSP [illegible]
(1) [illegible] accumulator [illegible] MAC-ה [illegible] product
[illegible] MAC 8-ו [illegible]
[illegible] overflow - ו [illegible]
(2) [illegible] optional scaling operator [illegible] MAC-ה [illegible]
[illegible] product [illegible] pipeline-ה [illegible]
(3) [illegible] overflow [illegible]
[illegible] MAC [illegible]

4.4 DFT

[illegible] DFT-ה FT [illegible] $T \equiv t_b - t_a$
[illegible] N [illegible] N [illegible]
[illegible] $t_n = t_a + n\Delta t$, $\Delta t \equiv \frac{T}{N}$
$t_a = t_0$ [illegible] $t_{n-1} = t_b - \Delta t$ [illegible] $t_{n-1} \approx t_b$ [illegible] $N \gg 1$ [illegible] $\Delta t \ll T$ [illegible] $t_n = n\Delta t$

- מרווח הדגימות Δt בין דגימות [illegible] — $t_0 = \frac{1}{f_0}$
- [illegible] — $\omega_k = k\Delta\omega$, $\Delta\omega = \frac{\Omega}{N}$
- [illegible] — $N\Delta\omega = \Omega = \frac{2\pi}{\Delta t}$, $\Delta\omega\Delta t = \frac{2\pi}{N}$
- $S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt \rightarrow S_k = \sum_{n=0} s_n e^{-i(k\Delta\omega)(n\Delta t)}$

נוסחת DFT:

- $S_k = \sum_{n=0}^{N-1} s_n e^{-i\frac{2\pi kn}{N}} = \sum_{n=0}^{N-1} s_n W_N^{kn} \quad (k=0,\dots,N-1)$

ה-Power spectrum = $|S_k|^2$ [illegible]

Nth root of unity:

- $W_N \equiv e^{-i\frac{2\pi}{N}} = \cos(\frac{2\pi}{N}) - i\sin(\frac{2\pi}{N})$ $\quad W_2 = e^{-i\frac{2\pi}{2}} = -1$

[illegible]

$S = W_N s$

נוסחת IDFT:

- $s_n = \frac{1}{N}\sum_{k=0}^{N-1} S_k e^{+i\frac{2\pi kn}{N}} = \frac{1}{N}\sum_{k=0}^{N-1} S_k W_N^{-nk}$

$$W = \begin{pmatrix} W_N^{00} & W_N^{01} & W_N^{02} & 1 \\ W_N^{10} & W_N^{1} & W_N^{1} & W_N^{3} \\ & & & \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 \\ 1 & W_N^2 & W_N^4 & W_N^6 \\ 1 & & & \end{pmatrix}$$

N=2

$$W_{N=2} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} s$$

$$S_0 = \sum_{n=0}^{1} s_n W_2^0 = s_0 + s_1$$
$$S_1 = \sum_{n=0}^{1} s_n W_2^n = s_0 - s_1$$

[illegible] DFT.

14 FFT

אלגוריתמי DFT מהירים FFT. [illegible] radix-2 DIT ו- DIF [illegible] $N=2^m$ [illegible]
Radix-4, Split Radix, ו-FFT842 [illegible] radix-2 [illegible] mixed radix-4 [illegible]
prime factor [illegible] N [illegible] 2 [illegible]
FFT [illegible] FFT.

14.1 יישומי DFT

[illegible] DFT [illegible]
[illegible] N [illegible]
Hard real Time: [illegible]
Soft real Time: [illegible]

- [illegible] DFT [illegible]
  [illegible] buffer [illegible]

Double buffering: [illegible] 2 buffers [illegible] processing buffer [illegible]
ה-acquisition buffer [illegible]
ה-buffers [illegible]
[illegible] buffer [illegible]
[illegible] data-overrun [illegible] N [illegible]
[illegible] $T_{p=1}$ [illegible]
[illegible] N [illegible] buffer [illegible]
[illegible] superlinear [illegible] $T_{process} = qN^p$, $p>1$
[illegible] N [illegible] N [illegible]
data-overrun [illegible]

ה-FFT [illegible] DFT [illegible] N [illegible] 2
$N=2^k$ [illegible] $O(N\log_2 N)$ [illegible] superlinear [illegible] FFT
[illegible] $N\rightarrow\infty$ [illegible]
[illegible] N [illegible] FFT [illegible] $\frac{N}{\log_2 N}$
$N=2^4=16$ [illegible] FFT [illegible] $N=2^{10}=1024$ [illegible]
[illegible]

- החיסכון הבסיסי והמרכזי: ה-FFT הוא בעצם סיבוכיות ה-N^2 כתוצאה מה DFT הישיר אם נחלק את הבעיה לשני [illegible] ... DFT באורך N ידרוש N^2 [illegible] ... 2 DFTs באורך $\frac{N}{2}$ ידרוש $2(\frac{N}{2})^2 = \frac{N^2}{2}$ [illegible] (עם ה-+ [illegible] ה- $\frac{N}{2}$ [illegible]).

4.2 [illegible]

[illegible] 1: סיבוכיות [illegible] N [illegible] N-1 [illegible] ... ב- $\frac{N}{2}$ [illegible] ... "[illegible]" [illegible] "[illegible]" [illegible] $\frac{N}{2}$ [illegible] ... $\frac{N}{2}$ [illegible] ... $\frac{N}{2}$ [illegible].

[illegible]: in-place [illegible] n [illegible]

[illegible] 2: [illegible] 2 [illegible] 2 [illegible] N+1 , A,B [illegible] ... N^2 [illegible] ... $\frac{N}{2}$ [illegible] ... $\frac{N}{2}-1$ [illegible].

$$A = A_L 2^{\frac{N}{2}} + A_R$$
$$B = B_L 2^{\frac{N}{2}} + B_R$$
$$C = AB = A_L B_L 2^N + (A_L B_R + A_R B_L) 2^{\frac{N}{2}} + A_R B_R$$
$$= A_L B_L (2^N + 2^{\frac{N}{2}}) + (A_L - A_R)(B_R - B_L) 2^{\frac{N}{2}} + A_R B_R (2^{\frac{N}{2}} + 1)$$

[illegible] 4 [illegible] 3 [illegible] N^2 [illegible] $3(\frac{N}{2})^2 = \frac{3}{4}N^2$ [illegible] 25%
[illegible] ... (2 [illegible]) ... $\log_2 N$ [illegible]: $O(N^{1.585}) \approx O(N^{\log_2 3})$.
[illegible] Tom-Cook [illegible] 2-ה [illegible] ... $O(N \log(N) 2^{[illegible]})$ [illegible] ... FFT-ב [illegible] $O(n \log n)$.

4.3 Derivation of the DIT FFT

[illegible] $O(N^2)$ [illegible] : 2 [illegible] ⇐ $\frac{N^2}{2} = (\frac{N}{2})^2 + (\frac{N}{2})^2$ [illegible] 2-ל [illegible] 2 [illegible] ... Partition ו- Decimation.

Partition: [illegible]
[illegible]

$\{x_0, x_1, \ldots, x_{N-2}, x_{N-1}\}$
Low Partition $\{x_0, x_1, \ldots, x_{\frac{N}{2}-1}\}$; High Partition $\{x_{\frac{N}{2}}, \ldots, x_{N-1}\}$

Decimation: [illegible]
[illegible]

$\{x_0, x_1, \ldots, x_{N-2}, x_{N-1}\}$
Even Decimation $\{x_0, x_2, \ldots, x_{N-2}\}$; Odd Decimation $\{x_1, x_3, \ldots, x_{N-1}\}$

- Decimation [illegible] Partition [illegible] ; [illegible].
- Partition [illegible] Decimation [illegible].
- 2 [illegible] FFT [illegible].

[illegible] $\frac{N}{2}$ [illegible] ... 2 [illegible] ... $\frac{N}{2}$ [illegible] $\frac{N}{2}$ [illegible]
[illegible] Partition [illegible] ... [illegible] 2Hz [illegible] ...

[illegible]:

- $W_N^N = 1$
- $W_N^{\frac{N}{2}} = -1$
- $W_N^2 = W_{\frac{N}{2}} \Rightarrow W_N = e^{-j\frac{2\pi}{N}}, \; W_{\frac{N}{2}} = e^{-j\frac{2\pi}{N/2}} = e^{-j\frac{2\pi \cdot 2}{N}} = (e^{-j\frac{2\pi}{N}})^2 = W_N^2$

יתרון ה-FFT נובע מהסימטריה של DFT: [illegible] ו-X_k ו-$X_{k+\frac{N}{2}}$ [illegible]:

- $X_k = \sum_{n=0}^{N-1} x_n W_N^{nk}$, • $X_{k+\frac{N}{2}} = \sum_{n=0}^{N-1} x_n W_N^{nk} \cdot W_N^{\frac{Nn}{2}} = \sum_{n=0}^{N-1} x_n W_N^{nk} (-1)^n$

נפתח אלגוריתם ה-DIT [illegible] לזוגיים ואי-זוגיים:

- $X_k = \sum_{n=0}^{N-1} x_n W_N^{nk} = \sum_{n=0}^{\frac{N}{2}-1} \left(x_{2n} W_N^{2nk} + x_{2n+1} W_N^{(2n+1)k} \right) = \underbrace{\sum_{n=0}^{\frac{N}{2}-1} x_n^E W_{\frac{N}{2}}^{nk}}_{\text{DFT של הזוגיים}} + \underbrace{W_N^k \sum_{n=0}^{\frac{N}{2}-1} x_n^O W_{\frac{N}{2}}^{nk}}_{\text{DFT של האי-זוגיים}}$

$x_n^E = x_{2n}$, $x_n^O = x_{2n+1}$, $n = 0,1,\ldots,\frac{N}{2}-1$ — [illegible] DFT [illegible]

- $x_{k+\frac{N}{2}} = \sum_{n=0}^{\frac{N}{2}-1} x_n^E W_{\frac{N}{2}}^{nk} - W_N^k \sum_{n=0}^{\frac{N}{2}-1} x_n^O W_{\frac{N}{2}}^{nk}$

[illegible] X_k ו-$X_{k+\frac{N}{2}}$ ← [illegible]
[illegible] ב-partitions [illegible]
[illegible]:

- $X_k = X_k^E + W_N^k X_k^O$, $k \le \frac{N}{2}-1$ • $X_{k+\frac{N}{2}} = X_k^E - W_N^k X_k^O$

$\frac{N^2}{4}$, $\frac{N^2}{4} + O(N)$ [illegible]

[illegible] פרפר הבסיס. W_N^k נקרא twiddle factor:

ה-FFT [illegible] של פרפר DIT
[illegible] FFT.

- [illegible] $(N-1)^2$ [illegible] ו-$N \cdot N-1$ [illegible]
  [illegible] 2 DFT $\frac{N}{2}$-point [illegible] (twiddle factor [illegible]) [illegible]:
  [illegible]: $2\left(\frac{N}{2}-1\right)^2+1 = \frac{N^2}{2} - 2N + 3$ [illegible]: $2\left(\frac{N}{2}\left(\frac{N}{2}-1\right)\right)+2 = \frac{N^2}{2} - N + 2$
  [illegible] $\frac{N}{2}$ [illegible]
  [illegible] $\frac{N^2}{2} + O\left(\frac{N}{2}\right)$ [illegible] $O(N)$ [illegible]
  [illegible] X_k^O ו-X_k^E [illegible] DFT [illegible]
  X_k^E [illegible] הפרפר:

[illegible] DFT N-point [illegible]
[illegible] הפרפר [illegible] DFT ה-
[illegible]
[illegible] $N=8$ [illegible] 543-545

- ה-N [illegible] DFT N-point: [illegible] $N=2^m$ [illegible] 2,
  [illegible] $m = \log_2 N$ [illegible] $\frac{N}{2}$ [illegible]
  [illegible] 2 [illegible] $\frac{N}{2}\log_2 N$ [illegible] $N\log_2 N$ [illegible]
  [illegible] 2 [illegible] 4 [illegible]
  [illegible].

$O(N\log_2 N)$

<u>[illegible] DIT FFT:</u>

[illegible] R-radix [illegible] DIT FFT [illegible] $\log_R N$ [illegible] $\frac{N}{R}$ [illegible]
[illegible] R [illegible] radix-2 [illegible] (4,4)
ה-DIT FFT [illegible] in-place [illegible]
[illegible]
[illegible]
ה-successive in-place decimations [illegible]
[illegible] in-place decimation [illegible]
[illegible] 545 [illegible]
[illegible]
[illegible] 545.
[illegible] in-place decimation [illegible] 2^m [illegible] bit-reversal ←
[illegible] bit-reversal [illegible]
[illegible]
[illegible] bit-reversed [illegible].

4.4 אלגוריתמי FFT נוספים

אלגוריתם DIT radix-2 ידוע כ-Cooley-Tukey. [illegible] $O(N\log N)$,
[illegible] W_N^{nk}, [illegible] N הוא highly composite.

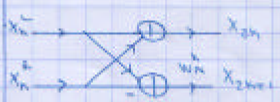
DIF: האלגוריתם דומה ל-DIT, [illegible]:

$x_n^L = x_n$, $x_n^R = x_{n+\frac{N}{2}}$ $\quad n = 0,1,\dots,\frac{N}{2}-1$

ע"י עיבוד ה-DIF נקבל:

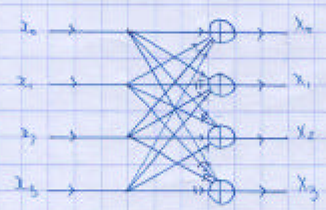
- $X_{2k} = (x_n^L + x_n^R)$
- $X_{2k+1} = (x_n^L - x_n^R)W_N^n$

כמו ב-DIT [illegible] partitions [illegible] DIF [illegible] twiddle factor [illegible] bit-reversal.

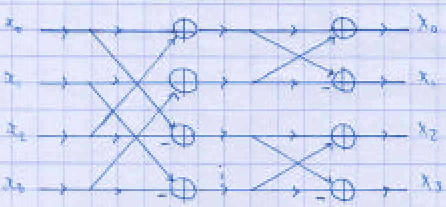


DIT radix-4: [illegible] N [illegible] 4 [illegible]
[illegible] 4-point DFT [illegible]

- $x_{4j}, x_{4j+1}, x_{4j+2}, x_{4j+3}$ $\quad (j=0\dots\frac{N}{4}-1)$
- $X_0 = x_0 + x_1 + x_2 + x_3$
- $X_1 = x_0 - ix_1 - x_2 + ix_3$
- $X_2 = x_0 - x_1 + x_2 - x_3$
- $X_3 = x_0 + ix_1 - x_2 - ix_3$



[illegible] 4 [illegible]
[illegible] radix-2 [illegible] 8 [illegible]
[illegible] 4-point [illegible] FFT radix-2
[illegible] radix-4 [illegible] N [illegible] 8 [illegible]
DIT [illegible] twiddle factors [illegible] $O(\frac{N}{4}\log_2 N)$ [illegible] radix-4 [illegible] $\frac{3N}{4}$ [illegible] twiddle factors [illegible], [illegible] $N\log_2 N - \frac{3}{8}\log_2 N$ [illegible] $\frac{3}{8}N\log_2 N$ [illegible]



[illegible] radix-4 [illegible] radix-2 [illegible] 8 [illegible] 2 [illegible]
[illegible] 4 [illegible] 4, X [illegible] 2 [illegible] radix-4 [illegible] radix-2 [illegible]
FFT742 [illegible] FFT842 [illegible] radix-8 [illegible]
[illegible] radix-4 [illegible] radix-2 [illegible]

split-radix: [illegible] DIF radix-2 [illegible] X_{2k+1} [illegible] twiddle factor [illegible] FFT [illegible] 2^m [illegible] FFT-[illegible] 2^{m-1} [illegible] twiddle factors [illegible] radix-2 [illegible] radix-4 [illegible] L [illegible]

• [illegible] 2 [illegible] 2 [illegible] zero-padding [illegible] FFT [illegible] zero-padding [illegible] 2 [illegible] split-radix [illegible] FFT842 [illegible] spectral points [illegible] spectral analysis [illegible] data points [illegible] FFT count-to-power [illegible] N [illegible]
[illegible] $N=R^m$ [illegible] Radix-R [illegible] $N=N_1N_2$ [illegible] N_1 [illegible] N_2 [illegible] DFT [illegible] N [illegible] N_2 [illegible] N_1 [illegible] twiddle factors [illegible] N [illegible] DFT [illegible] N_2 [illegible] in-place [illegible] mixed-radix [illegible] N_1, N_2 [illegible] twiddle [illegible] FFT prime factor FFT [illegible] Winograd [illegible] prime factor [illegible] inplace [illegible] pipelined [illegible] power-of-two.

[illegible] FFT-[illegible]

Four-point FFT: [illegible] 0 [illegible] 3 [illegible] 1, 2 [illegible]

$$\begin{pmatrix}X_0\\X_1\\X_2\\X_3\end{pmatrix}=\begin{pmatrix}1&1&1&1\\1&-i&-1&i\\1&-1&1&-1\\1&i&-1&-i\end{pmatrix}\begin{pmatrix}x_0\\x_1\\x_2\\x_3\end{pmatrix}$$

[illegible]: DFT [illegible]

$$W_4=\begin{pmatrix}1&1&1&1\\1&-i&-1&i\\1&-1&1&-1\\1&i&-1&-i\end{pmatrix}=\begin{pmatrix}1&0&1&0\\0&1&0&-i\\1&0&-1&0\\0&1&0&i\end{pmatrix}\begin{pmatrix}1&0&1&0\\1&0&-1&0\\0&1&0&1\\0&1&0&-1\end{pmatrix}$$

$x=W_4x$ [illegible] $4^2=16$ [illegible] x [illegible] 8 [illegible] 6 [illegible] bit-reversal [illegible]

$$W_4=\begin{pmatrix}1&0&1&0\\0&1&0&-i\\1&0&-1&0\\0&1&0&i\end{pmatrix}\begin{pmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&1&-1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}:\quad \underline{W_4}=\begin{pmatrix}\underline{I_2}&\underline{T_2}\\\underline{I_2}&-\underline{T_2}\end{pmatrix}\begin{pmatrix}\underline{W_2}&\underline{O_2}\\\underline{O_2}&\underline{W_2}\end{pmatrix}C_4$$

[illegible] I_2 [illegible] 2x2, T_2 [illegible] 2x2 twiddle factors [illegible] ($w_4^0=1$, $w_4^1=-i$), $\underline{W_2}$ [illegible] two-point DFT [illegible], $\underline{O_2}$ [illegible] 2x2, C_4 [illegible] 4x4 column permutation [illegible] DFT-[illegible] bit-reversal [illegible]

הביטוי המטריצי הוא בלוק:

$$W_{2m} = \begin{pmatrix} I_m & T_m \\ I_m & -T_m \end{pmatrix} \begin{pmatrix} W_m & 0_m \\ 0_m & W_m \end{pmatrix} C_{2m}$$

T_m מטריצה אלכסונית $m \times m$ עם [illegible]

W_{2m}, ה-twiddle factors של DFT בגודל 2m.

[illegible] ה-DFT [illegible] radix-2 DIT. [illegible]

[illegible] column permutation [illegible] in-place [illegible]

[illegible] 2, ה-W_m [illegible] DFT [illegible]

[illegible] twiddle [illegible]

14.6 Practical Matters עמ' 554 - 558

Parseval's relation: [illegible] x_n [illegible] N^2 [illegible]

[illegible] value [illegible]

14.7 [illegible] עמ' 558-560

ה-FFT [illegible]

[illegible]

[illegible]

zoom-FFT, interpolation, zero-padding [illegible]

[illegible]

14.8 [illegible] Goertzel

ב-FFT [illegible]

[illegible]

[illegible] FFT [illegible]

[illegible]

[illegible] N [illegible]

[illegible]: [illegible]

DTMF: Dual Tone Multiple Frequency, [illegible] 3 [illegible]

[illegible] Hz [illegible] decode [illegible] digit [illegible] 8 [illegible]

[illegible] decode [illegible] tone DTMF [illegible] 8 [illegible] band-pass [illegible]

[illegible]

decode [illegible] (digit) [illegible] DFT-ב [illegible] Goertzel

[illegible] band-pass [illegible] FFT-ב [illegible] band-pass [illegible]

FFT [illegible]

[illegible] FFT-ה [illegible]

[illegible] W_N^{nk} [illegible] DFT-ה [illegible]

X_k-ל [illegible] N [illegible] N-1 [illegible] x_n [illegible]

[illegible] 2N [illegible] 2(N-1) [illegible] W_N^{k}

[illegible] twiddle factors [illegible] spectral component-ה [illegible]

k-ה, ה-DFT [illegible] $W \equiv W_N^{k} = e^{-j\frac{2\pi k}{N}}$

- $X_k = \sum_{n=0}^{N-1} x_n W_N^{nk} = x_0 + x_1 W + x_2 W^2 + \ldots x_{N-1} W^{N-1}$

[illegible]:

- $X_k = ((\ldots(x_{N-1} W + x_{N-2}) W + \ldots + x_2) W + x_1) W + x_0$

[illegible]

- Given: x_n for $n \leftarrow 0 \ldots N-1$

```
P_{N-1} <- x_{N-1}
for n <- N-1 down to 0
    P_n <- P_{n+1} W + x_n
X_k <- P_0
```

[illegible]

$V \equiv W^{-1} = W_N^{-k} = e^{+j\frac{2\pi k}{N}}$. [illegible] $V^N = 1$ [illegible]:

- $X_k = \sum_{n=0}^{N-1} x_n V^{N-n} = x_0 V^N + x_1 V^{N-1} + \ldots + x_{N-2} V^2 + x_{N-1} V$

[illegible] X_k [illegible]

phase factor [illegible] $X'_k = x_0 V^{N-1} + x_1 V^{N-2} + \ldots + x_{N-2} V + x_{N-1}$

[illegible]:

Given: x_n for $n = 0..N-1$

$P_0 \leftarrow x_0$

For $n \leftarrow 1$ to $N-1$

$P_n \leftarrow P_{n-1}V + x_n$

$X'_k \leftarrow P_{N-1}$

[illegible] X_k

[illegible] $N-1$ [illegible]

[illegible] $\frac{2\pi k}{N}$ [illegible] x_n [illegible]

[illegible] $4(N-1)$ [illegible] $2(N-1)$ [illegible] $3(N-1)$ [illegible] $5(N-1)$ [illegible]

[illegible]

[illegible] P [illegible]

[illegible] time lag [illegible] 2 [illegible]

[illegible] $P_n \leftarrow P_{n-1}V$

V_{n-1} [illegible] $P_{n-1}V$ [illegible]

[illegible] Q_n: $P_n = Q_n - WQ_{n-1}$

[illegible] Q [illegible]: $P_n \leftarrow x_n + P_{n-1}V$

$Q_n - WQ_{n-1} \leftarrow x_n + (Q_{n-1} - WQ_{n-2})V$

$Q_n \leftarrow x_n + (V+W)Q_{n-1} - (WV)Q_{n-2}$

$Q_n \leftarrow x_n + AQ_{n-1} - Q_{n-2}$ $\quad A = V+W = 2\cos\left(\frac{2\pi k}{N}\right)$

[illegible] A [illegible] Q_n [illegible]

[illegible]:

- Given: x_n for $n = 0..N-1$

$Q_{-1} \leftarrow 0$, $Q_{-2} \leftarrow 0$

$Q_0 \leftarrow x_0$

For $n \leftarrow 1$ to $N-1$

$Q_n \leftarrow x_n + AQ_{n-1} - Q_{n-2}$

$X'_k \leftarrow Q_{N-1} - WQ_{N-2}$

[illegible] $|X_k|^2 = Q^2_{N-1} + Q^2_{N-2} - AQ_{N-1}Q_{N-2}$

[illegible] single frequency dependent coefficient [illegible] A [illegible] 2 [illegible] Q

[illegible]

[illegible] Goertzel.

[illegible] N [illegible] $\frac{N}{2}$ [illegible]

[illegible] N [illegible]

[illegible] bandwidth [illegible] N [illegible]

FIFO FFT

[illegible] FFT [illegible]

[illegible] FFT [illegible] to slide

[illegible] $DFT(x_0 \ldots x_{N-1}), DFT(x_1 \ldots x_N)$

[illegible] FIFO FFT [illegible] x_n [illegible]

[illegible] moving-average [illegible]

- $A_m = \sum_{n=0}^{N-1} x_{m+n}$

[illegible] N [illegible] FIFO [illegible] N [illegible]

[illegible] FIFO [illegible] A_0 [illegible] N [illegible],

[illegible]:

- $A_{m+1} = A_m + x_{m+N} - x_m$

[illegible] A [illegible] A_0 [illegible] x_n can been [illegible]

[illegible] moving averages [illegible] FIR [illegible]

[illegible] FT [illegible]: $X_{km} = \sum_{n=0}^{N-1} x_{m+n} W_N^{nk}$

[illegible] 'read the data' [illegible]

[illegible] FFT [illegible] DFT [illegible] FFT [illegible]

[illegible] FFT:

- $X_{k,m+1} = X_{k,m} + (x_{m+N} - x_m) W_N^{-mk}$

[illegible]

[illegible] FIFOFFT [illegible] N [illegible]

[illegible] N^2 [illegible] $\frac{N}{2}\log_2 N$ [illegible] $N\log N$ [illegible]

FFT [illegible] $O(N^2)$ [illegible] $O(N\log N)$ [illegible] FFT [illegible] $\log_2 N$,

[illegible] FIFOFFT [illegible] delay [illegible]

FFT [illegible] input buffer [illegible] FFT

[illegible] FIFOFFT [illegible] FT [illegible]

[illegible] FFT [illegible]