2.1.1 Which of the following are *signals*? Explain which requirement of the definition is possibly violated and why it is acceptable or unacceptable to do so.

1. **the height of Mount Everest**
   This is a signal with no time dependence, although we should define it as zero both before Everest was created, and after it is destroyed.

2. $\left(\mathbf{e^{it} + e^{-it}}\right)$
   Although it doesn't seem so at first glance, this is a real-valued function since it is twice $\cos(t)$, and is hence admissable as a signal. However it has infinite energy, and so we must understand the true signal to look just like $\cos(t)$ over the observation interval, but to become zero far enough outside it.

3. **the price of a slice of pizza**
   This is a time-dependent function (at least when there is some inflation). Prices are not continuous variables, since they are in whole cents, and so the signal is quantized in amplitude but not sampled in time.

4. **the 'sinc' function** $\frac{\sin(\mathbf{t})}{\mathbf{t}}$
   This is a finite duration signal. Note that it is well defined at $t = 0$.

5. **Euler's totient function $\phi(\mathbf{n})$, the number of positive integers less than n having no proper divisors in common with n**
   This is a rather strange signal. First it is defined only for positive integers, and gives integers values, and so is a digital signal defined for $t > 0$. Were we to observe this signal without knowing its origin we would think it to be highly unpredictable, and so stochastic.

6. **the water level in a toilet's holding tank**
   This is a analog signal which is usually constant, but at random times can quickly drop to zero and slowly recover.

7. $\lfloor \mathbf{t} \rfloor$ **the greatest integer not exceeding** t
   Similarly to the previous question, this function also has sharp jumps but these *are* objectionable. In addition this function increases without limit. We would probably not consider this a signal.

8. **the position of the tip of a mosquito's wing**
   This requires a three dimensional representation, and we usually only allow one dimensional signals.

9. $\sqrt{\mathbf{t}}$
   We can assume that we are talking about the positive square root. This signal would be undefined for negative times, and so we must fix the definition, e.g. by taking it to be zero. It is also unbounded and thus has infinite energy, but it increases so slowly that this would not a problem in practice.

10. **the Dow Jones Industrial Average**
    This is a valid (and interesting) digital signal defined at closing of business each day.

11. $\sin(\frac{\mathbf{1}}{\mathbf{t}})$
    This candidate signal is not defined at $t = 0$, nor is it 'well-behaved' in the vicinity of $t = 0$. In fact, it requires infinite frequencies. The combination of these two problems would probably make us decide that y isn't a signal at all, and hence answer *no* to this question. This type of decision is not easy; we frequently use white noise and delta functions as signals, but I do not know of any 'physical world' applications of $\sin(\frac{1}{t})$.

12. **the size of water drops from a leaky faucet**
    This is a digital signal, although it is not immediately evident that the sampling frequency is constant. This signal is related to the logistics recursion of the next question.

13. **the sequence of values** $\mathbf{x_n}$ **in the interval** $[\mathbf{0} \ldots \mathbf{1}]$ **defined by the** ***logistics recursion*** $\mathbf{x_{n+1}} = \lambda \mathbf{x_n}(\mathbf{1} - \mathbf{x_n})$ **for** $\mathbf{0} \leq \lambda \leq \mathbf{4}$
    This is a digital signal and will be discussed at length in Section 5.5. There is no restriction on how a signal is created. In fact we will often generate signals in just such a recursive manner.